УСИЛИЯ В ДЮБЕЛИТЕ НА КОМБИНИРАНИ СТОМАНО-СТОМАНОБЕТОННИ ГРЕДИ ВСЛЕДСТВИЕ НА ПРЕДВАРИТЕЛНО НАПРЯГАНЕ

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FORCES IN THE SHEAR STUDS OF COMPOSITE BEAMS DUE TO PRESTRESS

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Abstract: *This study aims to verify the design rule of Eurocode 4 for the distribution of the slab/girder interface force due to prestress. A preliminary study was carried out, and suitable parameters for parametric study were identified. A number of data sets giving the natural shapes of the distributions for varying steel/concrete modular ratios, connection stiffnesses, section types and points of application of the prestress force were obtained. A trapezoidal simplified shape was decided upon.*

Key words: *composite beams, shear studs, prestress, force distribution*

1. INTRODUCTION

One of the problems encountered in design and construction of composite continuous bridges and reinforced concrete bridges is the control of cracking over the supports. Review of the treatment of this problem in different countries is given in [1] , page 245. There are several practicable techniques that can be used for crack control: 1. By applying prestress to the composite section (usually in the slab); 2. By jacking the supports, 3. By combination of prestressing and jacking, 4. Control of the crack width by longitudinal ordinary reinforcement. The use of prestress for crack control is relatively rare because of the associated high loss of prestress, but a rule for dealing with it has to be incorporated in the relevant Eurocode [2].

As prestress is applied in the slab at some position between two main girders some of the force is transmitted to the steel section via the shear connection. This interface force comes as an additional loading on the shear connection and should be accounted for in the design of the shear connection by adding it to the forces produced by the main vertical loading. The situation is presented in Fig.1 . The relevant clause of the Eurocode is also illustrated the figure and assumes a trapezoidal distribution with the following values of the two parameters, that define its shape:

$e_d = 2e_h$ - according to the rule of thumb for distribution of the point force at 45° angle

$c=b_{eff} / 2$ - a constant value of c regardless of the position of the prestress force

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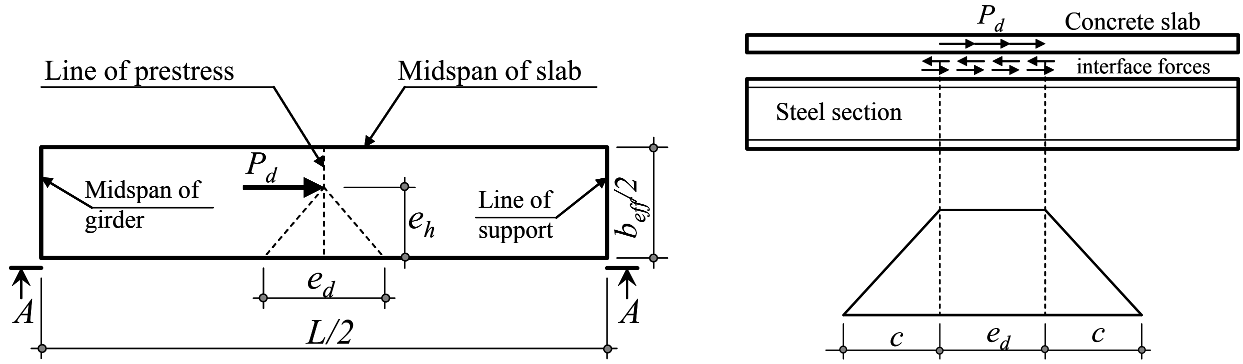

(a) Plan view *(b) Section A-A and distribution*

Fig. 1 *Overview of the problem*

The purpose of this study is to investigate the shape of the distribution by detailed analysis, and to propose a more accurate way of distributing the force among connectors close by.

2. FINITE ELEMENT MODELLING

One source was found describing modelling of a composite girder using a general program, [3], and another one using handmade program but again using general elements rather than specific ones incorporating composite behaviour [4]. Both models are two-dimensional. In the first, the plane of the model is the central plane of the slab. The steel section is represented by a set of springs and the shear studs are represented by another set of springs. The plane of the second model is the central plane of the web of the steel section. The steel section, the concrete slab and the shear stud have all been modeled by shell plane elements.

The engineering design package I-DEAS was used to carry out the modelling and analysis. The model used in this study is a 3-D one, where the slab is modeled by shell elements, the steel section is modeled by frame elements and the shear studs are modeled by cantilevers (frame elements) connecting the centre lines of the slab and the steel section, and having stiffness equivalent to the stiffness of the shear connection computed in the following way:

The end deflection of a cantilever loaded by a point load at the end is given by,

$$f = Pl^3/3EI \tag{1}$$

subsequently the stiffness of the cantilever (the force to produce a unit deflection) is:

$$3EI/l^3 \tag{2}$$

This has to be equal to the connection stiffness k :

$$k = 3EI/l^3 \tag{3}$$

Hence the second moment of area of a cantilever of length l in order to simulate a connector stiffness k has to be:

$$I = kl^3/3E \tag{4}$$

This formula has been used to calculate the second moments of area for the program input. The spacing of the cantilevers was initially set to span/40 , but this was found not to give enough data points, thus producing very ragged distribution which would be hard to investigate further. A spacing of span/200 was found convenient. Considering symmetry, the modelling region was set to a quarter of the area between two consecutive main girders and two consecutive internal supports. A force of 10kN was used throughout the study, since all analyses were carried out in the elastic range. The position of the applied prestress was considered to vary from straight above the studs, (load 0) to midspan of the slab (load 1), with four positions in between (load 0.125, load 0.25, load 0.5, load 0.75), the number signifying the relative distance to the shear studs. Before reaching the shape of the FE model given in Fig. 2, comparative studies were done using two other models: 1. beam model - same as the final model, except for the slab is modelled by beam element all along the length of the girder; 2. thin shell model - same as the final model except for the slab is modelled by thin shell elements all along the length of the girder. The beam model was found inadequate for carrying out detailed analysis, but could be used for finding some general trends in the behaviour at the verification and preliminary study stage. The thin shell model although providing the most aesthetic appearance and also the smoothest distributions was not used, because these advantages come at the expense of too many elements involved in the model. In the final model, the region close to the application point of the prestress force was modelled by shell elements and the rest of the slab by frame elements, as in the beam model. The two meshes were joined by appropriate constraints. The difference in the distributions in the final model and the shell model is given in Fig. 3

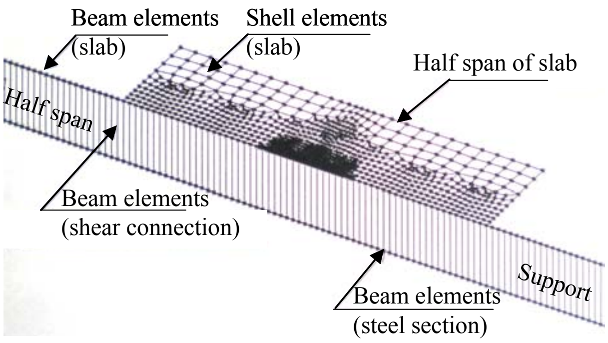

Fig. 2 *Mesh of the final FE model*

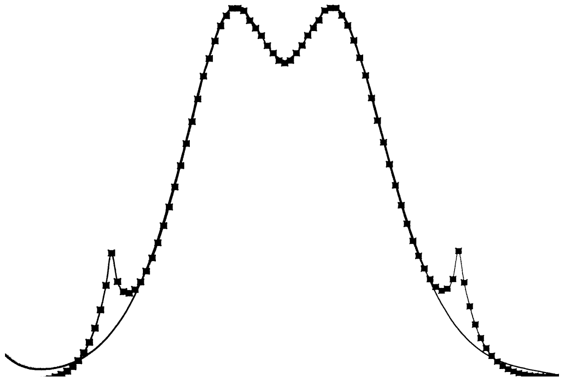

Fig. 3 *Distributions by the shell and the final model; solid line – shell model*

3. NUMERICAL ANALYSIS

Before going to the detailed parametric study, a thorough verification of the model was carried. This step included verification of the individual elements entering the model, verification of the behaviour as a general beam (very stiff connection), verification of the partial interaction behaviour, and boundary condition verification.

Next was a preliminary study, so as some general facts about the behaviour of the model could be established in order to narrow the scope of the parametric study. The following conclusions of general applicability could be made:

1. The distribution for non-uniform stiffness of connectors is the same as for a uniform distribution of the stiffness applicable to the vicinity of the prestressing force.
2. The shape of the distribution is dependant upon the stiffness of the shear connection giving steeper and narrower curves for stiffer connection.
3. The shape of the distributions remain the same regardless of the boundary conditions at midspan.
4. The distribution for the same section properties does not depend upon the length of the span producing absolutely identical distributions as long as the length is enough to allow for the distribution to develop.
5. The shape is also dependant upon the application point of the prestress producing steeper curves when the prestress gets closer to the longitudinal axis of the beam.

Next, the detailed parametric study was carried out. It consisted of a number of runs of the final model, representing various circumstances which could occur in a composite girder. Since the particular shapes of composite sections are finally defined by a series of design procedures, the geometrical and material parameters of the girders to be studied could not be varied at random. It was decided to choose cross sections that would represent the two distinctive types of composite plate girder bridges in terms of cross section geometry; the first type - with just two main girders and the second type - with more than two main girders. Density of the shear connectors - values of 30, 60, 120 No. 19mm studs / m were used. The properties of the two types of sections used are given in Table 1. Section #1 is of a relatively small size, and therefore will be used for shorter spans meaning the bridges made using it will be with more than two main girders. Section #2 is a big section, will be used for longer spans and the bridges made using it will be with two main girders.

Table 1 *Steel sections used for the parametric study*

section #	A_a [cm^2]	I_a [cm^4]	*h [cm]*	*d -12.5 [cm]*
1	405	608E+3	105.5	76.4
2	840	486E+4	210	154

Four types of girders were used, designated by the following syntax – type (section; b/2), where b/2 is in [cm]: A(1;175), B(1;250), C(2;250) and D(2;500). A total of eighteen runs were carried out as shown in Table 2. The names in the table have the following syntax: *girder type.connectors.concrete grade.concrete state*, and s. t. - short term; m. t. - medium term; l. t. - long term mean that the respective steel/concrete modular ratios apply.

Table 2. *Description of the model runs for the parametric study*

data set number	name	girder type	concrete grade	concrete state	No. 19 mm studs /m
1	A.60.40.st	A	C 40	s. t.	60
2	A.120.40.st	A	C 40	s. t.	120
3	A.30.50.st	A	C 50	s. t.	30
4	A.60.50.st	A	C 50	s. t.	60
5	A.120.50.st	A	C 50	s. t.	120
6	A.60.50.mt	A	C 50	m. t.	60
7	A.120.50.mt	A	C 50	m. t.	120
8	A.60.50.lt	A	C 50	l. t.	60
9	A.120.50.lt	A	C 50	l. t.	120
10	B.60.50.st	B	C 50	s. t.	60
11	B.60.50.mt	B	C 50	m. t.	60
12	B.60.50.lt	B	C 50	l. t.	60
13	C.60.50.st	C	C 50	s. t.	60
14	D.30.50.st	D	C 50	s. t.	30
15	D.60.50.st	D	C 50	s. t.	60
16	D.60.50.mt	D	C 50	m. t.	60
17	D.60.50.lt	D	C 50	l. t.	60
18	D.30.50.lt	D	C 50	l. t.	30

5. SUMMARY OF RESULTS AND DEVELOPMENT OF THE DESIGN RULE

Each of the analyses in Table 2 produced a distribution of the interface force. The distributions had either one peak – in the middle, or two peaks – as the one shown in Fig. 3. Some procedure had to be established in order to approximate these natural distributions with trapezoidal ones. The following was adopted:

1. For distributions with one peak, all values around the peak within 90% of the peak value were taken to constitute the plateau.

2. For distributions with two peaks all the values higher than the middle value (which lays in a valley) where chosen to constitute the plateau.

It was then a simple matter to compute the sloping parts of the distributions using the total interface force, which has to be the same for the natural as well as for the simplified distribution.

The most straightforward dimensionless parameters were thought to be the ratios e_h / e_d and 2*c* / *b*, which were adopted to describe the design rule. The ratio e_h / e_d appeared to vary very little among the analyses, so a constant value of e_h / e_d = 0.7 was adopted based on the grounds of not allowing *c* to become negative. The only dimensionless parameter that showed correlation with $2c / b$ was the ratio between the width of the slab and twice the distance between the centrelines of the steel section and the slab - $b / 2d$. This is not a parameter one would expect, but the reason for its applicability lies in the specific proportions of the composite girders. Thus it may be considered as an indirect parameter accumulating into itself some more complicated relationships that are hard to reveal.

One of the parameters of the distribution - e_h / e_d has already been set at a constant value of 0.7. The remaining task is to find a rule for calculating the second parameter of the distributions $2c / b$. The technique used to perform this task can be described as follows:

In the most general case the rule is sought in the form:

$2c / b = (2c / b)^{base} + k1(60 - N)/30 + k2(6.09 - b / 2d)/3.05 + k3(n-5.58)/11.178$

This equation is divided into two parts.

1. The dependence of $2c / b$ on $2e_h / b$. This relationship is called the base curve.

2. Added terms to take account of all or some of the additional parameters - the number of connectors, the modular ratio and the parameter $b / 2d$:

$k1(60 - N)/30$ - term accounting for the number of connectors per [m]

$k2(6.09 - b / 2d)/3.05$- term accounting for the ratio $b / 2d$

$k3(n-5.58)/11.178$ - term accounting for the modular ratio.

The parameter $2c / b$ was calculated for each data set, according to the assumed equation. For making this calculation possible, the parameters of the base curve and the coefficients were also assumed. The maximum force per unit length for each distribution was calculated.The percent error of the maximum force per unit length was calculated in the form: error % = 100*(calculated value - real value)/real value, meaning that negative values mean error on the unsafe side. A goal-seeking iteration procedure was employed to adjust the parameters of the base curve and the values of the coefficients, so as a minimum absolute value of the error for load position 0.125 can be achieved. The error was allowed to be either side (to safety or unsafety). Six versions of the design rule with varying complexity and accuracy were generated as shown in Table 3. Of all versions, Rule 6 does not incorporate the number of connectors per metre, has a linear base curve and error within 20% and is regarded as the best compromise between simplicity and accuracy.

Table 3. *Summary of the proposed design rules*

1.	description	Design rule incorporating the connection stiffness, the modular ratio and the parameter $b / 2d$
max error	variables involved	$2e_h / b, N, n, b / 2d$
[%]	base curve equation	$(2c / b) = 1.164 (e_h / b)^4 - 7.67 (2e_h / b)^3 + 8.96(2e_h /b)^2 - 1.44 (2e_h / b) + 0.0568$
10.24	full equation	base curve + $0.059 (60 - N) / 60 + 0.140(6.09-b/ 2d) / 3.05 + 0.0394 (n - 5.58) / 11.178$
2.	description	Design rule incorporating the parameter $b / 2d$
max error	variables involved	$2e_h / b , b / 2d$
[%]	base curve equation	$(2c / b) = 1.164 (e_h / b)^4 - 7.67 (2e_h / b)^3 + 8.96 (2e_h / b)^2 - 1.44 (2e_h / b) + 0.0568$
38.92	full equation	base curve + $0.235 (6.09 - b / 2d) / 3.05$
3.	description	Design rule without any additional parameters
max error	variables involved	$2e_h / b$
[%]	base curve equation	$(2c / b) = 4.94 (e_h / b)^4 - 17.12 (2e_h / b)^3 + 17.23(2e_h /b)^2 - 4.39 (2e_h / b) + 0.044$
34.79	full equation	base curve
4.	description	Design rule assuming linear relationship between $2c / b$ and $2e_h / b$, with no additional parameters

max error [%] 34.79	variables involved	$2e_h / b$
	base curve equation	$(2c / b) = 2.08\ (2e_h / b) - 0.16$
	full equation	base curve
5.	description	Design rule assuming linear relationship between $2c / b$ and $2e_h / b$, taking into account the modular ratio
max error [%] 34.79	variables involved	$2e_h / b, n$
	base curve equation	$(2c / b) = 2.15\ (2e_h / b) - 0.17$
	full equation	base curve + $0.05\ (n - 5.58) / 11.178$
6.	description	Design rule assuming linear relationship between $2c/b$ and $2e_h / b$, taking into account the modular ratio and the parameter $b / 2d$
max error [%] 18.62	variables involved	$2e_h / b, n, b / 2d$
	base curve equation	$(2c / b) = 1.70\ (2e_h / b) - 0.17$
	full equation	base curve + $0.13\ (6.09 - b / 2d) / 3.05 +$ $0.03\ (n - 5.58) / 11.178$

4. CONCLUSIONS

The design rule has been established studying the simplest possible type of composite girder - with a uniform plate steel section all along the span.

When plate girders of varying depth are used the rate of change of the depth and subsequently of the area properties of the section, is usually small. The rule can therefore be applied, and the value of the dimension *d* needed for calculating the sloping part *c*, should be taken as the value at the position along the span where the particular tendon is placed. It can be predicted, that in fact the distributions will not be symmetrical, but slightly tilted sideways to the direction of the decreasing section depth. The larger the length of the distribution ($2c + e_d$) the larger this effect will be. Since the length of the distributions decreases with the point of application of the prestressing force coming closer to the steel section, for the distributions at load positions close to load 0.125, which produce the largest interface forces per unit length, the tilting effect will be insignificant.

When open-top box girders are used, the rule can be applied regarding the steel section as consisting of two sections - as if the open-top section has been split into two sections along the middle of its bottom wall. The rule cannot be used for closed top box sections.

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